Black Scholes Pricing Models: An Innovative Ecologist Approach

KOH HOCK LYE[1], TEH SU YEAN[1], KEW LEE MING[1] and BAHARI BELATON[2]
[1]School of Mathematical Sciences
[2]School of Computer Sciences
Universiti Sains Malaysia, 11800 Penang, MALAYSIA.
hlkoh@cs.usm.my

Abstract - *The Black Scholes (BS) models for pricing derivatives have become a standard tool for evaluating derivatives for the past decades. The recent global financial tsunami and the US subprime crisis have, however, raised questions regarding the role of models in financial chaos and cast doubt concerning the reliability of models. The BS model is essentially a diffusion model that mimics many similar models used extensively in physics and ecology. Hence, insights and experience gained in these drift-diffusion-reaction models used in ecology to simulate ecosystem dynamics and stability are useful in developing innovative approaches and extensions to the classic BS models. The authors have developed a set of pricing models codenamed DOLPHIN to evaluate asset and derivative prices, improving upon the framework of BS models. This paper presents some results of Windows-based DOLPHIN simulations, drawing insights gained from previous ecosystem simulations and providing new understanding and approaches.*

Keywords: DOLPHIN, options, Black-Scholes models, derivatives.

1 Introduction: Diffusion in Ecology

Diffusion models have been extensively used in ecology and physics research in the past decades. A basic and popular approach is to begin with random walks and aggregate the outcomes over time scales and length scales that far exceed the scales of random walks. In one dimension, the random walk concept perceives an individual moving a short distance γ to either the right or the left in a short time τ in a completely random manner with even probability 0.5. Of course this presumed probability assumption may be readily relaxed. This type of movement is referred to as a random walk. These independent random walks continue over the next n steps. After a time interval of $n\tau$, this large group of individuals would spread out in some manner. What is the distribution of this group of individuals (particles) undergoing this random walk when n is large? What is the probability that a particle would arrive at a point r steps to the right or s steps to the left? It turns out that the classic Black Scholes models [1] and other models derived from them were developed essentially based upon similar concepts, perceiving investors as behaving in a manner similar to random walks, at least in theory. To what extent and under what situation would investors *misbehave* would ultimately determine the reliability and accuracy of these financial models. The position of the individual r steps to the right after a total of n steps might be achieved by moving p steps to the right and q steps to the left, in any combination of order, provided that p-q=r, p+q=n. Hence p= (n+r)/2 and q= (n-r)/2 (assuming momentarily that both numbers are integers). Application of calculus would arrive at the conclusion that the probability p(r,n) that a particle would arrive at a location r steps to the right after n steps is the Binomial or Bernoulli distribution,

$$p(r,n) = \left(\frac{1}{2}\right)^n \frac{n!}{\{(n+r)/2\}!\{(n-r)/2\}!}. \quad (1)$$

When n approaches infinity this binomial distribution converges to the Gaussian distribution (or the normal distribution) given below,

$$\lim_{n\to\infty} p(r,n) = \left(2/\pi n\right)^{1/2} \exp\left(-r^2/2n\right). \quad (2)$$

To convert this discrete formulation to a continuous model we set $x=r\gamma$ and $t=n\tau$ and regard (x,t) as continuous variables. Then the probability distribution of this continuous random walk is the normal distribution often encountered in probability theory and financial models based upon the BS world.

$$p(x,t) = \frac{1}{2\left(\pi Dt\right)^{1/2}} \exp\left(-x^2/4Dt\right). \quad (3)$$

This normal distribution may be verified to satisfy the basic diffusion model in the form of Partial Differential Equation (PDE) (4), which incidentally becomes the fundamental building blocks of the BS models.

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right). \quad (4)$$